

Research Article

ON THE FORM OF FORGETTING

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Abstract—Almost everyone would agree that the course of forgetting is some curvilinear function of time. The purpose of the research described herein was to identify the nature of that function. Three experiments are reported, two involving human subjects and one involving pigeons. The human experiments investigated this issue using recall of words and recognition of faces, whereas the pigeon experiment employed the standard delayed matching-to-sample task. In all cases, the course of forgetting was best described by a simple power function of time relative to five other reasonable alternatives (linear, exponential, exponential-power, hyperbolic, and logarithmic). Furthermore, a reanalysis of Ebbinghaus's (1885) classic savings function showed that it, too, declines as a power function of time. These findings suggest that the form of forgetting is a relatively robust property of memory performance and that its mathematical description, perhaps only coincidentally, matches that of the psychophysical function.

An important step in the evolution of any science, including the behavioral sciences, is the identification of lawful empirical regularities. Stevens's (1971) Power Law, which describes the relationship between stimulus intensity and subjective sensation, and Herrnstein's (1961, 1970) Matching Law, which describes the relationship between reinforcement and response allocation, represent two discoveries that spawned decades of productive research. In the present article, we investigate a possible empirical principle concerning the relationship between memory and time. More specifically, we ask whether the natural course of forgetting can be adequately characterized by a single mathematical function.

On virtually any memory task, a subject will remember less and less of what was learned as more and more time passes. Moreover, when performance is plotted as a function of time, the course of forgetting is not linear, but curvilinear. Klatzky (1980) commented that forgetting functions as diverse as those based on recall over a period of seconds and "savings" over a period of weeks appear remarkably similar. A rapid initial decline is usually followed by a long, slow decay. Similarly, in a recent survey of eyewitness memory experts, more than 80% of respondents agreed with the statement that "the rate of memory loss for an event is greatest right after the event, and then levels off over time" (Kassin, Ellsworth, & Smith, 1989).

The apparent similarity in the form of multifarious forgetting functions raises the question of whether a general mathematical law of forgetting can be formulated. Surprisingly, only a few studies concerned with this issue have ever been performed. Ebbinghaus (1885) suggested that his well-known savings func-

tion appeared to be logarithmic in form. More recently, Wickelgren found that forgetting functions produced by verbal recognition procedures were accurately described either by an exponential-power function (Wickelgren, 1972, 1974) or by a simple power function (Wickelgren, 1977). In the animal memory literature, White (1985) has repeatedly found that the simple exponential provides an acceptable fit to forgetting functions produced by the delayed matching-to-sample (DMTS) task. Harnett, McCarthy, and Davison (1984), however, suggested that the hyperbola seems to provide a more accurate description of forgetting on this task.

Table 1 lists the mathematical functions that have, at one time or another, been taken to represent the course of forgetting. For comparative purposes, the table also includes the equation for a straight line. As shown in the rightmost column, linear decay implies that the rate of change in the strength of the memory trace with respect to time, dy/dt , is constant. The second function, exponential decay, implies that the rate of forgetting slows as the strength of the memory trace declines (i.e., dy/dt is a constant proportion of y). The third function, hyperbolic decay, implies that the rate of forgetting decreases in proportion to the square of memory strength. The last three functions (logarithmic, power, and exponential-power) all imply, in one way or another, that the rate of forgetting is retarded by the passage of time. This property captures the essence of Wickelgren's (1972, 1974) trace-resistance theory of decay and is consistent with Jost's second law: "Given two associations of the same strength, but of different ages, the older falls off less rapidly in a given length of time" (Hovland, 1951).¹

The question under consideration here is whether one of the functions shown in Table 1 can consistently provide the

Table 1 Forgetting function candidates

Function	Equation	Differential Equation
Linear	$y = a - bt$	$dy/dt = -b$
Exponential	$y = ae^{-bt}$	$dy/dt = -by$
Hyperbolic	$y = 1/(a + bt)$	$dy/dt = -by^2$
Logarithmic	$y = a - b \log(t)$	$dy/dt = -b/t$
Power	$y = at^{-b}$	$dy/dt = -b(y/t)$
Exp-power	$y = ae^{-2b\sqrt{t}}$	$dy/dt = -b(y/\sqrt{t})$

¹ It should be noted that the power and logarithmic functions, both of which assume that dy/dt is an inverse function of time, are undefined at a retention interval of zero. In practice, if a retention interval of zero were employed, some modification of these functions would be needed. For example, the power function might be written as $a(t + 1)^{-b}$, which is defined at $t = 0$ and quickly begins to approximate t^{-b} as t increases.

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The Form of Forgetting

most accurate description of the course of forgetting. One issue that complicates the search for empirical regularity, however, concerns the nature of the measurement scale provided by the chosen dependent measure (such as proportion recalled). This issue has recently been the subject of a running debate in the memory literature over how to compare *rates* of forgetting, but exactly the same issue applies to an analysis of the mathematical form of forgetting (Bogartz, 1990, Loftus & Bamber, 1990, Slamecka, 1985, Wixted, 1990). Essentially, if the chosen dependent variable offers only a monotone measurement scale with respect to a psychological variable (such as memory trace strength), then any observed empirical regularities may or may not hold on the psychological level. Thus, for example, an empirical measure that decays as an exponential function might reflect a psychological variable that decays as a power function. The practical implication is that different empirical measures of memory may exhibit variability in the form of forgetting solely because they provide different measurement scales.

On the other hand, the possibility of empirical consistency across various measures of memory has never been seriously considered. If empirical inconsistency is the rule, then the search for regularity must move to another level (e.g., the psychological level). If, however, the form of empirical forgetting is not capricious and is instead consistent across variations in measures, tasks, and even species of animal, then the scaling issue will pose less of a problem. The research reported below investigated the mathematical form of forgetting functions with these considerations in mind. Specifically, three experiments involving widely different procedures were used to collect smooth forgetting curves amenable to mathematical analysis. The first experiment investigated the mathematical form of forgetting in a short-term free recall paradigm. The second involved long-term recognition of faces, and the third employed delayed matching-to-sample with pigeons. In addition, we consider the mathematical form of the well-known savings function reported by Ebbinghaus (1885) more than a century ago.

EXPERIMENT 1

Subjects

The subjects were eight undergraduate psychology students at the University of California, San Diego, who were recruited by on-campus bulletin board advertisement.

Materials and Design

The experiment employed a variation of the widely used Brown-Peterson task (Brown, 1958, Peterson & Peterson, 1959). Lists of six words were drawn randomly from a pool of 540 high-frequency nouns taken from Thorndike and Lorge (1944) and Kucera and Francis (1967). These lists served as the to-be-remembered material during experimental sessions. In addition, multiple lists of five nouns and adjectives of varying frequencies were selected from a separate pool of 600 words. These words served as distractors during the retention interval following the presentation of each target list. A different ran-

dom order of words (both target and distractor) was used for every subject.

The experimental design included two factors, both varied within subjects. The first factor, retention interval, was varied over five levels (2.5, 5, 10, 20, and 40 s), and the second factor, degree of learning, was varied over two levels (high and low). In each of seven sessions (one practice, six experimental), a subject studied and attempted to recall 15 target lists of six words each. The size of the retention interval in effect on a given trial was determined randomly with the restriction that each of the five intervals occur exactly three times within a session. Degree of learning was varied across sessions in a random order, with high and low degree of learning conditions in effect for three sessions each.

Procedure

The words in a target list were presented on a computer monitor, and the subject was instructed to read each word aloud as it appeared on the screen. The six target words were presented at a rate of two words per second in a vertical column at the center of the screen. In the high degree of learning condition, the six words remained on the screen for an additional 5 s while the subject continued to rehearse them aloud. In the low degree of learning condition, the words remained on the screen for an additional 1 s of rehearsal.

List presentation was always followed by a retention interval filled with a distractor task consisting of the overt reading and rehearsal of additional sets of five words. These distractor lists were presented in the same manner as the target lists and were each followed by 2.5 s of overt rehearsal (during which time the words were obscured with asterisks). Additional distractor lists were presented in the same manner until the retention interval was completed. At that point, the onset of recall was signaled by a series of question marks that appeared on the center of the screen. Subjects were allowed 60 s for written free recall following each list.

Results and Discussion

Figure 1 shows the average forgetting functions separately for the high (5-s) and low (1-s) degree of learning conditions. The data from the practice session were not included in this analysis, but all of the data from each experimental session were. Previous research on the rapid build-up of proactive interference (PI) on the Brown-Peterson task (Keppel & Underwood, 1962) suggests that data from the first few trials could be justifiably omitted from the analysis. However, in this experiment, performance on the first trial did not differ systematically from the level of performance on later trials, perhaps because PI reached maximal levels in the practice session. In light of this, and in order to obtain the smoothest possible empirical functions, the data from every experimental trial were included in the analysis.

The issue under investigation concerns the mathematical form of the forgetting functions shown in Figure 1. How well do the functions shown in Table 1 describe these forgetting functions? To answer this question, each function was fitted to th

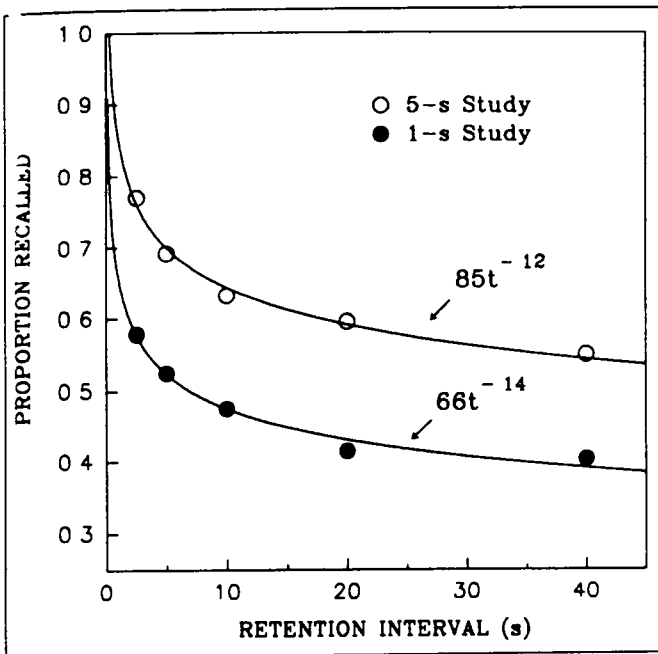


Fig 1 Proportion of words recalled for the high (5-s) and low (1-s) degree of learning conditions as a function of retention interval (Experiment 1) The solid curves represent the best-fitting power functions

data shown in Figure 1 using a nonlinear least-squares regression program, and the quality of the fit was calculated on the basis of the residual sum of squares (i.e., proportion of data variance accounted for). As shown in the first two columns of Table 2, the benchmark linear function accounts for a reasonable percentage of the data variance. Surprisingly, the exponential function, which describes so many natural processes from population growth to radioactive decay, did not improve much on the straight line. Similarly, the hyperbola, which describes the mathematical form of some learning curves (Mazur & Hastie, 1978), did not fare well here. The exponential-power function was reasonably accurate, and both the logarithmic and power functions were extremely accurate (with the edge going to the latter). The solid curves in Figure 1 represent the best-fitting power function to the high and low degree of learning curves.

EXPERIMENT 2

The results of the first experiment suggest that the course of forgetting might be described by a simple power function (or, possibly, a logarithmic function) of time. Alternatively, the result might hold only for the specific procedural and measurement details employed in that experiment. Therefore, the second experiment investigated the form of forgetting using different to-be-remembered material (faces instead of words), a different kind of memory test (recognition instead of recall), and much longer retention intervals (hours and days instead of seconds).

Subjects

The subjects were 195 undergraduates of the University of California, San Diego, who were enrolled in an introductory psychology course. Participation in the experiment satisfied a course requirement.

Materials and Design

Two factors were varied between subjects, retention interval varied over four levels (1 hour, 1 day, 1 week, and 2 weeks), and duration of exposure to each face, varied over two (11 vs 3 s each). Subjects were randomly assigned to eight groups formed by the factorial combination of retention interval and duration of exposure.

Procedure

During the study phase, the subjects examined 40 color slides of male faces for either 3 s each or 11 s each. Following the retention interval, they were shown a set of 80 color slides of faces, half of which they had seen in the first set. Each of the 80 test slides remained on the screen for 30 s, and the subject simply responded "yes" or "no" to indicate whether or not the face had been seen before. "Yes/No" response bias was controlled by telling all subjects that exactly half of the 80 slides were seen before.

Table 2 Mathematical descriptions of the data shown in Figures 1 through 3

Function	Word Recall		Face Recognition	DMTS	Ebbinghaus
	1 s	5 s			
Linear	74.4	77.5	64.5	78.6	44.8
Exponential	78.1	80.2	67.6	90.2	48.8
Hyperbolic	82.7	83.2	70.7	98.2	58.4
Exponential-Power	90.9	91.7	83.4	96.0	73.8
Logarithmic	96.8	97.8	97.0	92.5	94.6
Power	98.0	98.9	98.7	99.1	97.8

Note: The values represent the percentage of variance accounted for.

The Form of Forgetting

Results and Discussion

The proportion of correct "yes" responses (i.e., hits over hits plus false alarms) was subjected to an analysis similar to that of the previous experiment. Because the forgetting functions produced by the high and low degree of learning conditions were rather variable, however, the data were averaged across degree of learning in order to yield a smooth forgetting curve. An analysis of variance yielded main effects for degree of learning, $F(1,187) = 29.91, p < .01$, and retention interval, $F(3,187) = 4.60, p < .01$, but the interaction did not approach significance, $F(3,187) = 0.51$. Figure 2 shows the proportions of correct responses averaged over degree of learning.

The mathematical functions shown in Table 1, all of which decline to an asymptote of zero, cannot be directly fitted to the forgetting curve shown in Figure 2, which declines to a theoretical asymptote of 0.50 (the proportion of correct responses produced simply by guessing). To deal with this problem, the dependent measure may be transformed so that chance performance is represented by 0 rather than 0.50 (e.g., by using hits minus false alarms), or the mathematical functions themselves may be modified so that they approach an asymptote of 0.50 rather than 0. At least with regard to the present data, either approach yields the same result. We chose to leave the dependent measure untransformed and fit each mathematical function in the general form

$$p(c) = \frac{f(t) + 1}{f(t) + 2}, \quad (1)$$

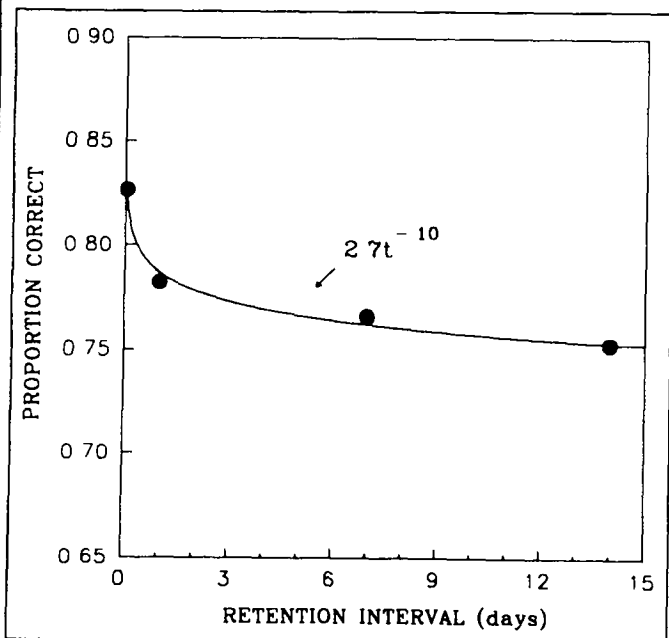


Fig 2 Hits divided by hits plus false alarms in face recognition judgments as a function of retention interval (Experiment 2). The solid curve represents the best-fitting power function (note the power function was fitted in the general form of $[f(t) + 1]/[f(t) + 2]$)

where $p(c)$ represents percent correct and $f(t)$ represents one of the mathematical functions shown in Table 1. An equation of this form results from the assumption that the familiarity of distractors remains constant over time and that subjects respond on the basis of the relative familiarity of old and new items. Expressed in this form, the mathematical functions now decline to the proper asymptote of 0.50 as $f(t)$ approaches zero.

As shown in the third column of Table 2, the pattern of results is strikingly similar to that observed in the previous experiment. The linear benchmark accounts for less than 75% of the variance, and the exponential and hyperbolic functions produce only marginal improvements. The exponential-power function accounts for a reasonable proportion of the data variance but, once again, the logarithmic and power functions account for nearly all of the variance (with the edge going to the latter).² The solid curve in Figure 2 represents the best-fitting power function.

EXPERIMENT 3

The results of the first two experiments point to the same conclusion. The course of forgetting is described by the simple power function (or, perhaps, by the logarithmic function). The fact that virtually identical conclusions were reached using such different procedures suggests that the identified form of forgetting may not be restricted to the procedural details of a particular experiment. To further evaluate this assumption, the next experiment involved a different species (pigeons) performing on yet another memory procedure, the DMTS task.

Subjects

The subjects were four experimentally naive White Carneaux pigeons maintained at approximately 80% of their free-feeding weight. They were housed in a vivarium in which lighting and ambient temperature were controlled automatically.

Materials and Apparatus

The pigeons were tested in a conditioning chamber equipped with three response windows mounted side by side on one wall. A high-resolution color graphics computer monitor was situated outside of the chamber directly facing the response windows. By looking through those windows, the pigeons were able to view graphic images displayed on the screen. Two graphic stimuli were used throughout the experiment, a red circle and a green square.

Procedure

A trial began with the display of one of the two stimuli in the center window. After 10 responses to the center window, the stimulus was extinguished and the retention interval began.

2. Fitting unmodified functions to hits minus false alarms or to d' did not alter the pattern of results shown in Table 2.

Upon completion of the retention interval, the red circle appeared in one side window and the green square in the other. A response to the stimulus that matched the previous stimulus on the center window was reinforced with two 45-mg food pellets. A response to the nonmatching stimulus terminated the trial. Retention intervals of 0.5, 1, 2, or 6 s were scheduled randomly within a session, and a 15-s intertrial interval was in effect throughout. After learning the task, the pigeons were run on this procedure for 15 sessions.

Results and Discussion

Figure 3 shows the forgetting functions produced by the four pigeons on this procedure averaged over the last five sessions. One pigeon developed a pronounced and apparently superstitious response bias at the 6-s delay only (an extreme avoidance of the right window). This point, which was actually slightly below chance, was excluded from the analysis.

As in the previous experiment, the dependent measure of interest is the proportion of correct responses, which declines to a theoretical asymptote of 0.50. Thus, the mathematical functions were placed in the form of Equation 1 in order to provide an appropriate fit. As shown in the fourth column of Table 2, all of the functions except a straight line provide a reasonable description of the data. Once again, the power function provides the most accurate description, but the hyperbola performs almost as well. Indeed, earlier work suggesting that the form of forgetting in pigeons is hyperbolic in nature (Harnett et al.,

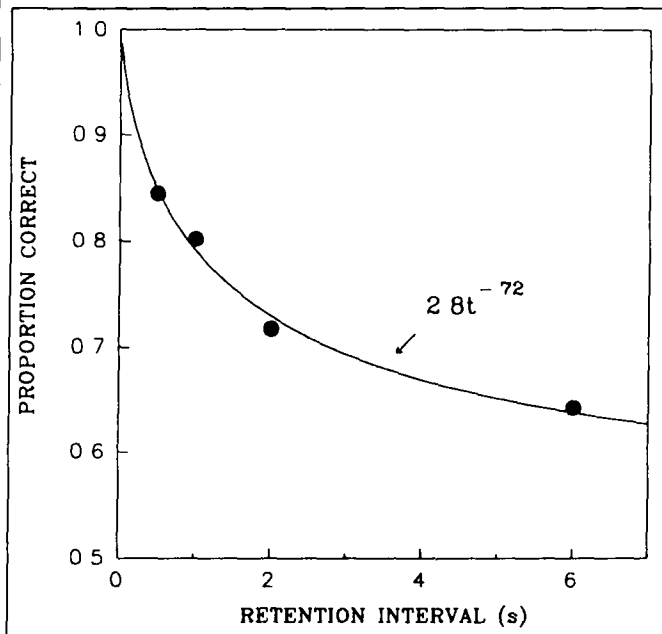


Fig 3 Proportion of correct responses as a function of retention interval by pigeons working on the delayed matching-to-sample task (Experiment 3). The solid curve represents the best-fitting power function (note the power function was fitted in the general form of $[f(t) + 1]/[f(t) + 2]$)

1984) is clearly not contradicted by the present results.³ The solid curve in Figure 3 represents the best-fitting power function.

EBBINGHAUS'S SAVINGS FUNCTION

Ebbinghaus (1885), who inaugurated so many important avenues of memory research, was also the first (apparently) to consider the mathematical form of forgetting. For that reason, it seems appropriate to consider his data here as well. Ebbinghaus's famous "savings" function, which is reproduced in almost every memory text, appeared to him to be logarithmic in form. The dependent variable, savings, is a measure of how much study was required to relearn a list of 13 nonsense syllables following a particular retention interval. A 50% savings, for example, indicates that the second learning of the list required only half the number of trials needed to learn the list initially.

Ebbinghaus did not consider how well other functions might describe his data, but we may do so here. Because the savings measure declines to a theoretical asymptote of zero, the functions shown in Table 1, may be fit in their standard form. The fifth column of Table 2 reveals that the pattern of results obtained in Experiments 1 and 2 was reproduced exactly here. The exponential and hyperbolic functions offered only a modest improvement over the straight line, while the logarithmic and power functions accounted for nearly all of the variance.

DISCUSSION

The purpose of this research was to determine whether memory performance is sufficiently stable across procedural and measurement variations to permit the identification of an empirical principle of forgetting. Although a definitive statement on this matter would be premature, the present results appear to favor the power function. Whether we consider recall, recognition, or savings as measures of memory, present words, faces, nonsense syllables, or graphic images as to-be-remembered material, use pigeons or humans as subjects, or employ seconds, days, or weeks as retention intervals, the result is the same. Forgetting proceeds as a simple power function of time.

The Scope of the Power Function

Although the power function has been shown to apply to an impressive array of memory paradigms, it seems important to bear in mind that few laws, even those in the physical sciences, are universally valid. The present results do not provide any specific information about the potential limits of the power function, but we should perhaps identify a few possibilities. First, and most obviously, the results may not hold outside the range of values used in the present experiments. With regard to the dependent measure, we did not obtain functions in the range above 90% correct. Forgetting functions falling in this range

³ Indeed, when the apparently aberrant data point is included in the analysis, the hyperbola slightly outperforms the power function (99.7% vs 99.0% VAF, respectively).

The Form of Forgetting

might not conform to a power law (and surely would not as performance approached the ceiling) With regard to the independent measure, the power function will necessarily become less and less accurate as the retention interval approaches zero (at which point the power function is undefined) In addition, the use of retention intervals of months or even years may yield results different from those observed in the present series of experiments Nevertheless, at least within the range of parameters considered here, the most common measures of forgetting appear to decline as a power function of time

Scaling Considerations

Earlier, we noted that disagreements about the mathematical form of forgetting might arise simply because of differences in the nature of the scale associated with each measure That is, if each dependent variable is assumed to provide an index of "trace strength" (or some retrieval process analogous to it), they might nevertheless exhibit differences in the form of forgetting merely because they provide different nonlinear measurement scales Given that the various measures instead appear to agree about the form of forgetting, what may we conclude about the decay of the theoretical memory trace?

The answer to this question would seem to depend on whether or not the relevant theoretical construct is taken to represent a real, fundamental, and potentially measurable entity (e.g., the neural basis of memory) If so, it would be hard to justify an assumption that our behavioral measures, no matter how consistent, can anticipate the form of decay according to measures taken on another level Although neural processes may indeed behave in accordance with the power law, a definitive answer will presumably remain unknown until the relevant measurement technology is developed

On the other hand, if our theories are construed as a mechanism by which to organize existing facts and draw valid inferences about the world (e.g., van Fraassen, 1980), then, because a different standard applies, we arrive at a different conclusion If every reasonable behavioral measure of memory conforms to a power law of forgetting, then theories that incorporate this assumption about theoretical memory processes will most parsimoniously guide our understanding of the empirical world Thus, until a more compelling alternative is advanced, memory can be reasonably represented as a process that decays or weakens as a power function of time

Does this point of view also imply that our empirical measures provide a linear scale of the strength of the theoretical memory trace? Not necessarily, and the reason for this is most easily seen by examining the relationship between two different empirical measures of memory Consider, for example, an experiment in which the recall forgetting function, R , declines as $R = a_1 t^{-b_1}$ and the savings forgetting function, S , as $S = a_2 t^{-b_2}$ The relationship between S and R is found by solving each of these functions for t , setting the resulting equations equal to each other, and solving for R as a function of S Following these steps yields $R = a_1 (S/a_2)^{b_1/b_2}$ Thus, if the decay constants (b_1 and b_2) happen to be identical, then recall is a linear function of savings (at least over the range of values considered) If the decay constants are not equal, however,

then the relationship between R and S is nonlinear and is itself described by a power function In the same way, the relationship between any dependent measure and the theoretical memory trace may be nonlinear even though both may be described by a function of the general form, at^{-b}

These considerations also bear on how memory might be expected to decay according to another common measure of memory, d' When experimental conditions are arranged so as to keep response bias constant (as was true of Experiment 2), then the relationship between proportion correct, $p(c)$, and d' is clearly curvilinear (Swets, Tanner, & Birdsall, 1964) If $p(c)$ declines as a power function of time, what does this imply about decay according to d' ? As illustrated above, that depends on the nature of the curvilinear relationship between the two measures At least over the range of values obtained in Experiment 2, the curvilinear relationship between $p(c)$ and d' is reasonably well-described by a power function Thus, when d' is used as the dependent measure instead of $p(c)$ in Experiment 2, the pattern of results is virtually identical to that shown in Table 2 (with the power function accounting for 99.99% of the variance) Perhaps it is not surprising, then, that Wickelgren (1977) found that his verbal recognition data, expressed as d' , were accurately described by the power function as well Thus, while $p(c)$ and d' may not necessarily always agree about the form of forgetting, as a general rule, both decline as a power function of time

Memory and Psychophysics

We conclude by noting that the function that seems to describe the course of forgetting has a long and well-established history in psychology The field of psychophysics has for more than a century been concerned with the mathematical function that describes the relationship between subjective sensation, S , and the intensity of the physical stimulation, I Indeed, Fechner's Law stated that the function was logarithmic in form $S = k \log(I)$ While this function reigned supreme for many years, it was eventually replaced by a power function, $S = kI^b$, which, of course, has come to be known as Stevens' Power Law Although skeptics persist to this day, the power function seems to capture the psychophysics of many physical dimensions, including loudness, brightness, temperature, weight, and time According to Stevens, each of these dimensions has its own characteristic exponent (Stevens, 1971) For judgments of loudness, the exponent is usually about 0.67 For judgments of brightness, the exponent is closer to 0.33

The apparent similarity in the mathematical form of forgetting functions and psychophysical functions may be merely coincidental or may instead reflect a more fundamental connection between the two fields For example, if we assume that the subjective intensity of the memory trace declines as a function of time in the same way that the subjective intensity of a light would diminish as a function of distance, then the power law of forgetting can readily be derived from the psychophysical function Indeed, White (1991) recently proposed a model based on the assumption that the dynamics of memory and psychophysics are, in principle, the same

If this idea is correct, then the discovery of additional sim

similarities between the two fields would not be altogether surprising. For example, just as the psychophysical exponent is determined by the nature of the perceived physical dimension, we may find that the forgetting function exponent is largely determined by the nature of the to-be-remembered material (e.g., faces, meaningful words, nonsense syllables, etc.). Such findings would establish a degree of empirical regularity one step beyond the mathematical form of the forgetting function, and the search for such regularity represents a potentially useful extension of the work reported here. For the present, however, our conclusions are restricted to the relationship between memory and time. Forgetting, like subjective sensation, appears to obey a power law.

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